To solve this problem, we first need to construct a 99 percent confidence interval for the proportion of vaccine-eligible people in the United States who received the flu vaccine. We are given a sample size \( n = 2350 \) and the observed number of successes (people who received the vaccine) \( x = 978 \).

The sample proportion \( \hat{p} \) is calculated as follows:

\[

\hat{p} = \frac{x}{n} = \frac{978}{2350} \approx 0.4166

\]

For a 99 percent confidence interval, the critical value \( z^\* \) for a normal distribution is approximately 2.576.

The standard error (SE) for the sample proportion is:

\[

SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.4166 \times (1-0.4166)}{2350}} \approx 0.0101

\]

The margin of error (ME) is then:

\[

ME = z^\* \times SE = 2.576 \times 0.0101 \approx 0.0260

\]

Thus, the 99 percent confidence interval is:

\[

\hat{p} \pm ME = 0.4166 \pm 0.0260

\]

This gives us a confidence interval of:

\[

(0.3906, 0.4426)

\]

Since the interval \((0.3906, 0.4426)\) does not contain 0.45, we have evidence to suggest that the belief that 45 percent of vaccine-eligible people had received the flu vaccine may not be accurate.

Next, we will determine the smallest sample size needed for the Canadian survey to ensure that the margin of error is less than or equal to 0.02 with a 99 percent confidence level.

The formula for margin of error is given by:

\[

ME = z^\* \times \sqrt{\frac{p\_0(1-p\_0)}{n}}

\]

where \( p\_0 \) is a prior estimate for the population proportion. We generally use 0.5 for \( p\_0 \) when no prior estimate is available, as this maximizes the product \( p\_0(1-p\_0) \) and yields the most conservative (largest) sample size.

Rearranging the margin of error formula to solve for \( n \) gives:

\[

n = \left(\frac{z^\*}{ME}\right)^2 \times p\_0(1-p\_0)

\]

Substitute the given values:

\[

n = \left(\frac{2.576}{0.02}\right)^2 \times 0.5 \times (1-0.5) = \left(\frac{2.576}{0.02}\right)^2 \times 0.25

\]

\[

n = (128.8)^2 \times 0.25 = 16555.84

\]

Thus, the smallest sample size that can be used to guarantee that the margin of error will be less than or equal to 0.02 is \( \lceil 16555.84 \rceil = 16556 \). Therefore, the recommended sample size is 16,556.